IB EAST Honors Geometry
Algebra Review Packet

There are 8 sections that you need to finish by the first day of class. You need to have the packet done by August 10th. All problems must be done on ***separate paper***, not on these worksheets. It must be done legibly. All problems must include work and checks as demonstrated in the examples. Please include the cover sheet that is on the back of the letter you received with this packet. Answers are provided for each problem on the last sheet of the packet. ***YOU WILL BE TESTED ON THIS MATERIAL ON THE FIRST FULL DAY OF CLASSES WITHOUT REVIEW!*** This is not a short assignment; don't wait until the end of the summer. Prepare for the test on the first full day of class. If you have questions about the assignment, not the material, you may email me, joy.smith-wallace@polk-fl.net.

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1. Solving Linear Equations

After completing Algebra I, you should have a good grasp of solving linear equations. Therefore, there is no instruction, only some comments. Solving linear equations is really the process of unwrapping a variable found in an equation. You unwrap it using the reverse order of operations.

Order of Operations:

1. Grouping Symbols including fraction bars
2. Exponents/Radicals
3. Multiplication/Division from left to right
4. Addition/Subtraction from left to right

Before you unwrap the variable, you may need to simplify both sides of the equation and get the variable on only one side of the equation.

Look at the complicated equation to the right as an example.	$2(x - 5) - 4x - 17 = 3x - 2(x + 3)$
First, distribute.	$2x - 10 - 4x - 17 = 3x - 2x - 6$
Second, combine like terms.	$-2x - 27 = x - 6$
Next, get the x's on the left side by subtracting 1x from both sides	$-3x - 27 = -6$
Unwrap the variable. Add 27 to both sides.	$-3x = 21$
Continuing to unwrap the variable by dividing both sides by -3.	$x = -7$

These problems are easy to check. You need only put the value you found for x into the original equation to see if it the left side is equal to the right side.

$$2(x - 5) - 4x - 17 = 3x - 2(x + 3)$$
$$2((-7) - 5) - 4(-7) - 17 = 3(-7) - 2((-7) + 3)$$
$$2(-12) + 28 - 17 = -21 - 2(-4)$$
$$-24 + 28 - 17 = -21 + 8$$
$$-13 = -13 \checkmark$$

Since it checks, we are confident we have the correct answer.

Solve these linear equations and check your answers on a separate sheet of paper. Show your work for the solution and the check. Answers can be found at the end of this packet. For additional help, check out www.MrPelzer.com.

1. $2x + 5 = 11$
2. $3x + 5 = -16$
3. $2(x - 3) = 84$
4. $5x - 32 = 80$
5. $3(2x + 5) - 3x = 6$
6. $2(4 - 2x) - 3(2x - 9) = 125$
7. $3x - 4(x - 4) + 4 = 13$
8. $3(2x - 9) + 6x = -19$
9. $2(4x - 3) + 4 = 5x - 6$
10. $x - 3(x - 7) = 4(x - 7) - 2x$
11. $3(4 - 2(5x - 3) - 2x) = 8(x + 1)$
12. $5(4(3 - 2(x - 1)) + 2) = 2(55 - x)$

2. Solving Systems of Equations by <u>Linear Combination Method</u>

(also called Elimination Method)

This method of solving a system of equations adds two equations (or multiples of the equations) together to create an equation with only one variable which can be solved using properties of algebra. The equations must be in standard form, Ax + By = C.

Example 1: Find the values of x and y in the system. $\begin{cases} 4x - 3y = 11 \\ 3x + 2y = -13 \end{cases}$

If we add the two equations together, we will get a new equation. When we do this we want one of the variables to be eliminated. If we add the equations the way they are now, this will not happen.

$$\begin{array}{r} 4x - 3y = 11 \\ +\ 3x + 2y = -13 \\ \hline 7x - y = -2 \end{array}$$

We can choose either variable to eliminate, so in this problem we will eliminate the x. The x coefficients (now 4 and 3) must be opposites so when they are added to together, they cancel each other out leaving us with 0. We will change them both into their least common multiple of 12. To do this, multiple both sides of the top equation by 3 and the bottom equation by -4.

$$\begin{cases} (3)(4\text{x} - 3\text{y}) = (11)(3) \\ (-4)(3\text{x} + 2\text{y}) = (-13)(-4) \end{cases}$$

$$\begin{cases} 12\text{x} - 9\text{y} = 33 \\ \text{-}12\text{x} - 8y = 52 \end{cases}$$

Adding them this time, the variable x cancels out.

$$\begin{array}{r} 12x - 9y = 33 \\ +\ -12x - 8y = 52 \\ \hline -17y = 85 \end{array}$$

We can solve this linear equation by dividing both sides of the equation by -17 and find the value of y.

y = -5

To find the value of x, we can do it two different ways:

Method I:
Eliminate the y variable in the same way. The least common multiple for the coefficients of y is 6, so multiply the top equation by 2 and the bottom equation by 3 (note one is already negative and the other positive)

$$\begin{cases} (2)(4\text{x} - 3\text{y}) = (11)(2) \\ (3)(3\text{x} + 2\text{y}) = (-13)(3) \end{cases}$$

$$\begin{cases} 8\text{x} - 6\text{y} = 22 \\ 9\text{x} + 6y = -39 \end{cases}$$

$$\begin{array}{r} 8x - 6y = 22 \\ +\ 9x + 6y = -39 \\ \hline 17x = -17 \end{array}$$

Dividing both sides by 17, we get x = 1.

x = -1

Method II:
Plug the value -5 in for y in either of the two equations and solve for x.

Using top equation, (bottom would work, too) we subtract 45 from both sides, and then divide by 12 on both sides to find x.

4x – 3(-5) = 11
4x + 15 = 11
4x = -4
x = -1

The solution is x = -1 and y = -5. Write this in an ordered pair, (-1, -5).

Of course, these answers should be checked in both equations.

$$\begin{array}{cc} 4(-1) - 3(-5) = 11 & 3(-1) + 2(-5) = -13 \\ -4 + 15 = 11 & -3 + -10 = -13 \\ 11 = 11 \checkmark & 13 = -13 \checkmark \end{array}$$

When you solve a system using the linear combination of method, choose the variable you want to eliminate wisely. Sometimes you won't have to change either equations or only one to get one of the variables to have opposite coefficients.

Since these are linear equations you can graph them. If you get a solution, then the two lines will intersect at the coordinates of the solution. But some lines do not intersect, but are parallel. When you solve a system of parallel lines, both variables will be eliminated simultaneously and what will remain will not be true.

$$\begin{cases} 2x - 3y = 6 \\ 2x - 3y = 4 \end{cases}$$

$$\begin{array}{r} 2x - 3y = 6 \\ + -2x + 3y = -4 \\ \hline 0 = 2 \end{array}$$

When solving other systems, the variables may both be eliminated at the same time and what remains is true. In this case, the two equations you had in your system were actually the same line. There are infinite solutions, any x and y value that are on the graph of the line.

$$\begin{cases} 2x - 3y = 6 \\ 2x - 3y = 6 \end{cases}$$

$$\begin{array}{r} 2x - 3y = 6 \\ + -2x + 3y = -6 \\ \hline 0 = 0 \end{array}$$

Therefore, when solving a system of equations both variables are eliminated, if what remains is true, you have infinite solutions because both equations are of the same line. If what remains is not true, then you have no solutions since the equations are of parallel lines.

Solve each system for x and y by linear combination (elimination) on a separate sheet of paper. Check your answers. Show your work for the solution and the check. Answers can be found at the end of this packet. For additional help, check out www.MrPelzer.com.

1. $\begin{cases} 2x - 3y = 2 \\ 5x - 3y = 14 \end{cases}$

2. $\begin{cases} 2x + 7y = 3 \\ -4x - 2y = -18 \end{cases}$

3. $\begin{cases} 8x - 6y = 16 \\ 12x - 9y = 24 \end{cases}$

4. $\begin{cases} 51x + 17y = 102 \\ 3x - 5y = -66 \end{cases}$

5. $\begin{cases} x - y = 39 \\ x + y = 1785 \end{cases}$

6. $\begin{cases} 32x + 24y = 42 \\ 36x + 27y = -26 \end{cases}$

7. $\begin{cases} 6x + 4y = 7 \\ 15x - 12y = 1 \end{cases}$

8. $\begin{cases} 11x - 3y = -39 \\ 6x + 12y = -19 \end{cases}$

9. $\begin{cases} -3x - 10y = 9 \\ 14x + 18y = -22 \end{cases}$

10. $\begin{cases} 2y - 3x = 51 \\ 11x - 7y = -187 \end{cases}$

11. $\begin{cases} 2x - 3y = 4y - 3x - 68 \\ x - 4y = 3x + 3y - 61 \end{cases}$

12. $\begin{cases} 4(x + 3y) + 25 = -3(2x - y) + 3 \\ 5x - 3(x - 2y) = 2(y - 6) + 1 \end{cases}$

3. Solving Systems of Equations by ***Substitution***

This method of solving a system of equations solves for one of the variables in one of the two equations and then substitutes this value for the same variable in the other equation. This creates an equation with only one variable which can be solved using properties of algebra.

Example 1: Find the values of x and y for the system. $\begin{cases} 4x - 3y = 14 \\ 3x + y = 17 \end{cases}$

We can choose any variable in the system to solve for, but all will involve fractions except the y in the second equation. To get the y alone on one side of the equation we subtract 3x from both sides of the equation to get y = -3x + 17.

$$y = -3x + 17$$

$$4x - 3y = 14$$
$$4x - 3(-3x + 17) = 14$$
$$4x + 9x - 51 = 14$$
$$13x - 51 = 14$$
$$13x = 65$$
$$x = 5$$

Substitute this value into the other equation, we will get an equation involving only the x variable. We can solve this equation to find the value of x. First, we distribute the -3, combine like terms, add 51 to both sides, and then divide both sides by 13 to determine x = 5.

To find the value of x, we could substitute the value of x into either of the two original equations to find the value of y, but the easiest equation to use to find y is the one circled, the one we solved for the variable y. Substituting 5 in for x we apply the order of operations to determine that y = 2. The solution is therefore x = 5 and y = 2, which should be written as an order pair (5, 2).

$$y = -3x + 17$$
$$y = -3(5) + 17$$
$$y = -15 + 17$$
$$y = 2$$

Of course, these answers should be checked in both equations.

4(5) – 3(2) = 14	3(5) + (2) = 17
20 – 6 = 14	15 + 2 = 17
14 = 14 ✓	17 = 17 ✓

As with solving other systems with linear combination (elimination), the variables may both be eliminated when substituting. In this case if what remains is true, you have infinite solutions because both equations are of the same line. If what remains is false, then you have no solutions since the equations are of parallel lines. In the example to the right, since what remains is false, there are no solutions – graphing both linear equations would give you parallel lines.

$$\begin{cases} 2x - y = 6 \\ 2x - y = 8 \end{cases}$$
$$y = 2x - 6$$
$$2x - (2x - 6) = 8$$
$$2x - 2x + 6 = 8$$
$$6 \neq 8$$

Substitution is the preferred method for solving a system of equations when you can easily solve for a variable. If you cannot do this without creating fractions, the preferred method for solving a system of equations is linear combination (elimination).

Another example: $\begin{cases} 8x - 5y = 19 \\ 6x + 2y = -4(x+1) + 8 \end{cases}$

First put the second equation into standard form by distributing the 4 and then moving the 4x over to the other side by subtracting it from both sides.

$$\begin{aligned} 6x + 2y &= -4(x+1) + 8 \\ 6x + 2y &= -4x - 4 + 8 \\ 6x + 2y &= -4x + 4 \\ 10x + 2y &= 4 \end{aligned}$$

This gives us a system of equations in standard form. There is no variable with a coefficient with 1 or -1 and none of the equations have a GCF that could change this. but looking at the equations, the second equation has a GCF of 2 for all the terms. Dividing both sides of the equation by 2, we get a different version of this system.

$$\begin{cases} 8x - 5y = 19 \\ 10x + 2y = 4 \end{cases}$$

$$\begin{cases} 8x - 5y = 19 \\ 5x + y = 2 \end{cases}$$

Solving for y in the second equation, we get y = -5x + 2. This can be substituted into the first equation to get:

$$\begin{aligned} 8x - 5(-5x + 2) &= 19 \\ 8x + 25x - 10 &= 19 \\ 33x - 10 &= 19 \\ 33x &= 29 \\ x &= \frac{29}{33} \end{aligned}$$

$$\begin{aligned} y &= -5\left(\frac{29}{33}\right) + 2 \\ y &= \frac{-145}{33} + \frac{66}{33} \\ y &= \frac{-79}{33} \end{aligned}$$

Check the solution $\left(\frac{29}{33}, \frac{-79}{33}\right)$.

$$\begin{aligned} 8\left(\frac{29}{33}\right) - 5\left(\frac{-79}{33}\right) &= 19 \\ \frac{232}{33} + \frac{395}{33} &= 19 \\ \frac{627}{33} &= 19 \\ 19 &= 19 \; \checkmark \end{aligned}$$

$$\begin{aligned} 6\left(\frac{29}{33}\right) + 2\left(\frac{-79}{33}\right) &= -4\left(\left(\frac{29}{33}\right) + 1\right) + 8 \\ \frac{174}{33} + \frac{-158}{33} &= -4\left(\frac{62}{33}\right) + 8 \\ \frac{16}{33} &= \frac{-248}{33} + \frac{264}{33} \\ \frac{16}{33} &= \frac{16}{33} \; \checkmark \end{aligned}$$

Solve each system for x and y using substitution on a separate sheet of paper. Check your answers. Show your work for the solution and the check. Answers can be found in the end of the packet. For additional help, check out www.MrPelzer.com.

1. $\begin{cases} 2x + y = 21 \\ 7x - 2y = 90 \end{cases}$

2. $\begin{cases} 2x + 7y = -20 \\ x - 5y = -10 \end{cases}$

3. $\begin{cases} x - 6y = -2 \\ -5x + 30y = 10 \end{cases}$

4. $\begin{cases} 2x - 4y = 40 \\ 8x - 3y = 82 \end{cases}$

5. $\begin{cases} 9x - 2y = -6 \\ 5x + 4y = 12 \end{cases}$

6. $\begin{cases} 2x + 3y = 8 \\ 9x - 3y = 14 \end{cases}$

7. $\begin{cases} 2(x - 3y) = x - 76 \\ 5(x + 2y) = 1 - 3(x - 6y + 43) \end{cases}$

8. $\begin{cases} 10x - 5y = 3 \\ 6x + 30y = 81 \end{cases}$

9. $\begin{cases} 2x - 6y = -3 \\ 10x - 6y = -5 \end{cases}$

4. Simplifying Radicals

There are three rules that must be followed for a radical to be considered in simplest form. There are two properties of radicals that allow us to follow those rules. They are:

Multiplication Property: $\sqrt{ab} = \sqrt{a}\sqrt{b}$ Division Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

4
9
16
25
36
49
64
81
100
121
144
169
196
225
256
289
324
361
400

The three rules are:

1. No perfect-square factors allowed inside a radical.
2. No fractions allowed in a radical.
3. No radicals allowed in the denominator of a fraction.

If an answer to a problem involves a radical, it may need to be simplified.

Since 12 is not a perfect, we need to find any perfect square factors (not including 1) of 12. The factor in this case is 4. We can change 12 into 4 x 3 and by the Multiplication Property it changes $\sqrt{4 \cdot 3}$ into $\sqrt{4} \cdot \sqrt{3}$, which simplifies to $2\sqrt{3}$. This is the simplified form of $\sqrt{12}$.

$$\sqrt{12}$$
$$\sqrt{4 \cdot 3}$$
$$\sqrt{4}\sqrt{3}$$
$$2\sqrt{3}$$

More examples:

$$5\sqrt{72}$$
$$5\sqrt{36}\sqrt{2}$$
$$5 \cdot 6\sqrt{2}$$
$$30\sqrt{2}$$

$$5\sqrt{72}$$
$$5\sqrt{9}\sqrt{8}$$
$$5\sqrt{9}\sqrt{4}\sqrt{2}$$
$$5 \cdot 3 \cdot 2\sqrt{2}$$
$$30\sqrt{2}$$

$$\sqrt{1200}$$
$$\sqrt{100}\sqrt{12}$$
$$\sqrt{100}\sqrt{4}\sqrt{3}$$
$$10 \cdot 2\sqrt{2}$$
$$20\sqrt{2}$$

The second rule is easy to satisfy. First reduce the fraction and then apply the quotient rule. This leads to the 3rd rule – no radicals in the denominator.

$$\sqrt{\frac{14}{6}} = \sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}}$$

Meeting this rule is done by multiplying by 1. Multiplying by 1 doesn't change its value, but it must be a special 1 to help. It must be a fraction with the same numerator and the denominator and both equal to the denominator of the original fraction. When you multiply it, the demonator will be the root of a perfect square. Then just simplify the denominator and reduce the fraction if necessary.

$$\frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{\sqrt{9}}$$

$$\frac{\sqrt{21}}{\sqrt{9}} = \frac{\sqrt{21}}{3}$$

More examples:

$$\sqrt{\frac{32}{10}} = \sqrt{\frac{16}{5}} = \frac{\sqrt{16}}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

$$\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{\sqrt{25}}$$

$$\frac{4\sqrt{5}}{\sqrt{25}} = \frac{4\sqrt{5}}{5}$$

$$\frac{4\sqrt{6}}{\sqrt{12}} = \frac{4\sqrt{1}}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

$$\frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{4}}$$

$$\frac{4\sqrt{2}}{\sqrt{4}} = \frac{4\sqrt{2}}{2} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

> You can cancel common factors if both numbers are in the radical or if both are outside the radical.

Note: to check if operations were done correctly, put the original problem and the answer into your calculator and you should get the same decimal approximations.

Simplify each radical on a separate sheet of paper. Show your work for the solution. Answers can be found in the end of the packet. For additional help, check out www.MrPelzer.com.

1. $\sqrt{40}$

2. $\sqrt{52}$

3. $\sqrt{80}$

4. $\sqrt{320}$

5. $\sqrt{243}$

6. $\sqrt{288}$

7. $\sqrt{675}$

8. $\sqrt{\frac{4}{9}}$

9. $\sqrt{\frac{3}{7}}$

10. $\sqrt{\frac{6}{10}}$

11. $\sqrt{\frac{12}{8}}$

12. $\frac{5}{\sqrt{15}}$

13. $\frac{3}{\sqrt{3}}$

14. $\frac{2\sqrt{3}}{\sqrt{5}}$

15. $\frac{3\sqrt{5}}{\sqrt{20}}$

16. $\sqrt{\frac{50}{75}}$

17. $\frac{16}{\sqrt{24}}$

18. $\frac{10\sqrt{10}}{\sqrt{80}}$

5. *Solving Quadratic Equations by the Quadratic Formula*

In Algebra 1, you were given 3 ways to solve a quadratic equation- graphing, factoring, and using the quadratic equation. We will use the last 2 methods extensively in Honors Geometry. Let's start with the quadratic formula. This is a formula that if a quadratic equation is in standard form will give you the solution to the quadratic equation. You will learn in Algebra 2 how the formula is derived by using another method of solving quadratic equations called Completing the Square. Keep in mind, because a quadratic equation is a second degree equation because largest exponent for a variable is 2, there will be 2 solutions.

standard form for quadratic equations: $ax^2 + bx + c = 0$

quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For an example, solve the equation to the right. $x^2 + 3x = 4$

First put this into standard form. $x^2 + 3x - 4 = 0$

Identify a, b, and c. $a = 1, b = 3, c = -4$

Plug these values into the quadratic equation and solve.

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)}$$

We will deal with expression inside the radical first. This expression, $b^2 - 4ac$, is known as the discriminant. In this problem, the value is 25, a perfect square. This means we will get a rational solution. If its value had been 0, then there would have been only one solution- a double solution. If its value had been positive but not a perfect square, then the 2 solutions would have been irrational. If the disriminant's value is negative, you will get 2 imaginary solutions – which you will learn about in Algebra 2.

$$x = \frac{-3 \pm \sqrt{9+16}}{2}$$

$$x = \frac{-3 \pm \sqrt{25}}{2}$$

$$x = \frac{-3 \pm 5}{2}$$

Once the discriminant is simplified, the formula is separated into 2 equations to find the two solutions.

$$x = \frac{-3+5}{2} \qquad x = \frac{-3-5}{2}$$

$$x = \frac{2}{2} \qquad x = \frac{-8}{2}$$

$$x = 1 \qquad x = -4$$

In this case, x = {-4, 1}.

Quadratic equations are easy to check. Substitute the both value of x you found into the equations and see if the left side is equal to the right side.

If x = -4,
$(-4)^2 + 3(-4) = 4$
$16 + -12 = 4$
$4 = 4$ ✓

If x = 1,
$(1)^2 + 3(1) = 4$
$1 + 3 = 4$
$4 = 4$ ✓

Since it checks, we are confident both answers are correct.

Sometimes, the discriminant is not a perfect square. In this case, the radical must simplified, but then you do not need to separate it into 2 different equations.

Solve $2(x^2 + 3x) = 1$

$2x^2 + 6x = 1$

$2x^2 + 6x - 1 = 0$

$a = 2$, $b = 6$, $c = -1$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36+8}}{4}$$

$$x = \frac{-6 \pm \sqrt{44}}{4}$$

$$x = \frac{-6 \pm 2\sqrt{11}}{4}$$

$$x = \frac{-3 \pm \sqrt{11}}{2}$$

Using a calculator, approximate answers can be found.

$$x = \frac{-3 + \sqrt{11}}{2} \approx 0.16, \quad x = \frac{-3 - \sqrt{11}}{2} \approx -3.16$$

To check these, it is quickest to use your calculator with the approximate answers. Your answers will not be exact, but should be close if correct.

$2((.16)^2 + 3(.16)) = 1$
$2(.0256 + .48) = 1$
$2(.5056) = 1$
$1.0112 \approx 1$ ✓

$2((-3.16)^2 + 3(-3.16)) = 1$
$2(9.9856 + -9.48) = 1$
$2(.5056) = 1$
$1.0112 \approx 1$ ✓

Solve each quadratic equation by the quadratic formula on a separate sheet of paper. Check your answers. Show your work for the solution and the check. Answers can be found in the end of the packet. For additional help, check out www.MrPelzer.com.

1. $x^2 + 5x + 4 = 0$
2. $x^2 - x = 6$
3. $x^2 = -6x$
4. $x^2 + 8 = 6x$
5. $x^2 + 3x = 12x - 1$
6. $-20 = x^2 + 5(2x + 1)$
7. $4x^2 + 8x + 2 = 0$
8. $4x^2 = 4x - 1$
9. $-8x + 3x^2 = -1$
10. $3a^2 + 6a + 2 = 0$
11. $x^2 + x + 1 = 0$
12. $4x^2 - 3x = 7$

6. *Solving Quadratic Equations by* ***Factoring*** *(when a=1)*

The second method for solving quadratic equations is through factoring. Unlike the quadratic formula, factoring will only give solutions that are rational, solutions that can be written as fractions. One advantage of factoring, is that it can be much faster then the quadratic formula. We start with quadratic equations with a leading coefficient of 1, where a = 1 when in standard form, $ax^2 + bx + c$. Factoring works because the equation of adding 3 terms is converted to an equation multiplying 2 binomials. The only way the product of the two quantities is 0, is if one of the factors is 0. So, by determining what makes each binomial equal to 0, we find the solutions.

Take the example at the right. First put the equation into standard form where the x^2 term is positive.

$$x^2 + 6x = -8$$

$$x^2 + 6x + 8 = 0$$

What we want to do is to convert this equation into 2 binomials being multiplied. If you FOIL to multiply these binomials, the Firsts gives you x^2, Outer and Inner give you qx and px, while the L give you pq. This means that pq = 8 and p + q = 6. So, we need to determine what factors of 8 will add up to 6.

$$(x + p)(x + q) = 0$$
$$x^2 + qx + px + pq = 0$$
$$x^2 + 6x + 8 = 0$$

Making a list of factors of 8, the only one that works is 2, 4.
Two binomials can be created using 2 and 4.

factors of 8	
1,8	2, 4
-1, -8	-2, -4

From this product, we know that if either one of the factors is 0, then the product will be 0. So this problem becomes 2 problems, each factor = 0. Solving both of the equations, we find that x = -4, -2.

$$(x + 2)(x + 4) = 0$$

$x + 2 = 0$ $\quad$ $x + 4 = 0$
$x = -2$ $\quad$ $x = -4$

As stated before, quadratic equations are easy to check. Substitute the both value of x you found into the equations and see if the left side is equal to the right side.

If x = -4,
$(-4)^2 + 6(-4) = -8$
$16 + -24 = -8$
$-8 = -8$ ✓

If x = -2,
$(-2)^2 + 6(-2) = -8$
$4 + -12 = -8$
$-8 = -8$ ✓

Since it checks, we are confident both answers are correct.

Here is one more example:

$x^2 – 12 = 4x$

$x^2 – 4x – 12 = 0$

Factors of 12	
3, -4	-3, 4
1, -12	-1, 12
2, -6	-2, 6

$(x + 2)(x – 6) = 0$

$x + 2 = 0$ $\quad$ $x – 6 = 0$
$x = - 2$ $\quad$ $x = 6$

therefore, x = {-2, 6}

$(-2)^2 – 12 = 4(-2)$
$4 – 12 = -8$
$-8=-8$ ✓

$(-6)^2 – 12 = 4(-6)$
$36 – 12 = -24$
$-24 = -24$ ✓

Solve each quadratic equation by factoring on a separate sheet of paper. Check your answers. Show your work for the solution and the check. Answers can be found in the end of the packet. For additional help, check out www.MrPelzer.com.

1. $x^2 + 4x + 3 = 0$
2. $x^2 - 2x - 35 = 0$
3. $x^2 - 8x + 12 = 0$
4. $x^2 - 10x - 24 = 0$
5. $x^2 = 2x + 48$
6. $x^2 - 9x = 2x + 12$
7. $x(x + 8) = 4(11 - 3x)$
8. $x^2 + 5x = 5(x + 5)$
9. $32x + 240 = -x^2$

7. Solving Quadratic Equations by <u>Factoring</u> (when $a \neq 1$)

Factoring is more difficult when a is not equal to 1. The goal is still to change the quadratic equation in standard form to the product of 2 binomials.

This will be factoring by grouping.

$$6x^2 + 6x = -5x - 4$$

First put the quadratic equation into standard form and identify a, b, and c.

$$6x^2 + 11x + 4 = 0$$

$$a = 6,\ b = 11,\ c = 4$$

Then multiply a and c to get the product of 24.

$$ac = 24$$

Now list the factors of this product (24).

<u>factors of 24</u>

1, 24	2, 12
-1, -24	-2, -12
3, 8	4, 6
-3, -8	-4, -6

Find the factors from the list that will add up to b (11). In this case, the factors are 3 and 8, since 3 + 8 = 11.

Next rewrite the linear factor using these 2 factors. So replace 11x with 8x + 3x. (3x + 8x will also work).

$$6x^2 + 8x + 3x + 4 = 0$$

Group the first two terms and the last two terms with parenthesis.

$$(6x^2 + 8x) + (3x + 4) = 0$$

Pull out the gcf from the grouped terms. If both of the terms in one of the groups are negative, pull the negative out with the gcf. If this method is done correctly, once the gcf's are pulled out, the remaining binomials will be the same. In this case, (3x + 4) is left from both groups. To finish factoring, pull out this same binomial.

$$2x(3x + 4) + 1(3x + 4) = 0$$

$$(3x + 4)(2x + 1) = 0$$

Setting each binomial factor equal to 0, solve the new equations to find the solution to our original equation. x = {-1/2, -4/3}.

$$3x + 4 = 0 \qquad 2x + 1 = 0$$

$$3x = -4 \qquad 2x = -1$$

$$x = -\frac{4}{3} \qquad x = -\frac{1}{2}$$

Let's check the solution

$$6\left(-\frac{4}{3}\right)^2 + 6\left(-\frac{4}{3}\right) = -5\left(-\frac{4}{3}\right) - 4$$

$$6\left(\frac{16}{9}\right) - \frac{24}{3} = \frac{20}{3} - 4$$

\

$$\frac{32}{3} - \frac{24}{3} = \frac{20}{3} - \frac{12}{3}$$

$$\frac{8}{3} = \frac{8}{3}\ ✓$$

$$6\left(-\frac{1}{2}\right)^2 + 6\left(-\frac{1}{2}\right) = -5\left(-\frac{1}{2}\right) - 4$$

$$6\left(\frac{1}{4}\right) - \frac{6}{2} = \frac{5}{2} - 4$$

$$\frac{3}{2} - \frac{6}{2} = \frac{5}{2} - \frac{8}{2}$$

$$-\frac{3}{2} = -\frac{3}{2}\ ✓$$

Note, checking with a TI-83 or TI-84 calculator is really easy. Ask how if you're interested.

Here's another example. Find the solutions of the equation. $8x^2 - 10x = 12$.

Put into standard form: $8x^2 - 10x - 12 = 0$

Identify a, b, and c: $a = 8$, $b = -10$, and $c = -12$

Find the product of a and c: $ac = (8)(-12) = -96$

List the factors of -96, noting that since b = -10, the negative factor will be the largest one

1, -96 2, -48 3, -32, 4, -24 **6, -16** 8, -12

If the equation is factorable, than one of these pairs must add up to -10. 6 and -16 are the ones.

Now rewrite $8x^2 - 10x - 12 = 0$ using these factors: $8x^2 + 6x - 16x - 12 = 0$

Group the first two terms and the last two terms: $(8x^2 + 6x) + (-16x - 12) = 0$
Note that since both terms in the 2nd grouping are negative, we will pull out a negative with the gcf.

Factor out the gcf's out of both groups: $2x(4x + 3) + -4(4x + 3) = 0$

Factor out the common binomial (4x + 3): $(4x + 3)(2x + -4) = 0$

Set each factor to 0 and solve for x.

$$2x - 4 = 0 \qquad 4x + 3 = 0$$
$$2x = 4 \qquad 4x = -3$$
$$x = 2 \qquad x = -\tfrac{3}{4}$$

We conclude that x = {- ¾, 2}.

Let's check the solution

$$8\left(-\frac{3}{4}\right)^2 - 10\left(-\frac{3}{4}\right) = 12$$
$$8\left(\frac{9}{16}\right) + \frac{30}{4} = 12$$
$$\frac{9}{2} + \frac{15}{2} = 12$$
$$\frac{24}{2} = 12$$
$$12 = 12 \checkmark$$

$$8(2)^2 - 10(2) = 12$$
$$8(4) - 20 = 12$$
$$32 - 20 = 12$$
$$12 = 12 \checkmark$$

The solution to this quadratic could have been a lot easier to find if we recognized that all the terms had a greatest common factor (GCF) of 2. If we divide both sides of the equation by 2, we get fewer factors to check.

$8x^2 - 10x = 12$

Divide both sides by the common factor: $4x^2 - 5x = 6$

Put into standard form: $4x^2 - 5x - 6 = 0$

Identify a, b, and c: $a = 4$, $b = -5$, and $c = -6$

Find the product of a and c: $ac = (4)(-6) = -24$

List the factors of -24, noting that since b = -5, the negative factor will be the largest one

1, -24   2, -12   3, -8   4, -6

If the equation is factorable, than one of these pairs must add up to -5. 3 and -8 are the ones.

Now rewrite $4x^2 - 5x - 6 = 0$ using these factors: $4x^2 + 3x - 8x - 6 = 0$

Group the first two terms and the last two terms: $(4x^2 + 3x) + (-8x - 6) = 0$

Factor out the gcf's out of both groups: $x(4x + 3) + -2(4x + 3) = 0$

Factor out the common binomial (4x + 3): $(4x + 3)(x + -2) = 0$

Set each factor to 0 and solve for x.

$4x + 3 = 0$
$4x = -3$
$x = -\frac{3}{4}$

$x -+ -2 = 0$
$x = 2$

We conclude that x = {- ¾, 2}, which we have already checked.

Always look for a common factor for all the terms. If you find one and divide it out on both sides, you will always find an easier equation to solve.

Solve each quadratic equation by factoring on a separate sheet of paper. Check your answers. Show your work for the solution and the check. Answers can be found in the end of the packet. For additional help, check out www.MrPelzer.com.

1. $3x^2 - 10x + 8 = 0$
2. $2x^2 + x - 3 = 0$
3. $5x^2 = 2x + 3$
4. $24x - 35 = 4x^2$
5. $-9x^2 + 15x = 4$
6. $6x^2 = 2(13x + 10)$
7. $12x^2 + 39x + 27 = 0$
8. $7x + 21 = 14x^2$
9. $-72x^2 + 36x + 36 = 0$

8. Solving Special Cases of Quadratic Equations

There are two special cases of the quadratic equations which have much easier solutions. They are when there are missing terms in the quadratic equation. The x^2 term must be present in order for the equation to be quadratic, but the linear term (x term) or the constant (number only) could be missing. In fact they are not really missing, the b = 0 when no linear term is present and c = 0 when we do see the constant. The manner in which we find solutions for each of these is simple.

First example involves no linear term.

$$3x^2 - 16 = 4$$

The goal is to isolate x. We need to get rid of the 16, 3 and the square. Using the reverse order of operations to solve an equation, eliminate subtracting 16 by adding 16 to both sides first. Second, undo multiplying by 3, by dividing both sides by 3. Lastly, to undo the exponent of 2, take the square root of both sides, remembering that when you add a square root to the equation, you must also add $\pm$ - which gives us our two answers.

$$3x^2 = 20$$

$$x^2 = \frac{20}{3}$$

$$x = \pm\sqrt{\frac{20}{3}}$$

Unfortunately, our answer is not simplified. It actually violates all the rules concerning radicals. Two steps are done right away, eliminating the fraction inside the radical and pulling out a perfect square of 4 from the 20.

$$x = \pm\frac{\sqrt{4}\sqrt{5}}{\sqrt{3}} = \pm\frac{2\sqrt{5}}{\sqrt{3}}$$

Next multiply by the special one and we get out final answer.

$$x = \pm\frac{2\sqrt{5}}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \pm\frac{2\sqrt{15}}{3}$$

Of course we should always check our answer.

$$3\left(\frac{2\sqrt{15}}{3}\right)^2 - 16 = 4$$

$$3\left(\frac{4*15}{9}\right) - 16 = 4$$

$$3\left(\frac{20}{3}\right) - 16 = 4$$

$$20 - 16 = 4$$

$$4 = 4 \checkmark$$

The second special quadratic equation has no constant term.

$$6x^2 - 11x = 4x$$

To find the solutions for this equation, first arrange the equation into standard form. Then pull out the GCF which will include an x. This creates a multiplication problem with two 1st degree factors, each of which can be set to 0 and solved. The GCF factor is the easiest and will always result in a solution of x = 0. The second will is an easy linear equation to solve.

$$6x^2 - 15x = 0$$

$$3x(2x - 5) = 0$$

$$3x = 0 \qquad x - 5 = 0$$

$$x = 0 \qquad 2x = 5$$

$$x = \frac{5}{2}$$

Please note that unlike a quadratic equation with all three terms present, we do not want to divide by the GCF to simplify the problem. We will lose a solution and when x does = 0, you would be dividing by 0 – a big no-no in mathematics.

Of course we check out solutions.

$$6(0)^2 - 11(0) = 4(0)$$
$$0 - 0 = 0$$
$$0=0$$

$$6(\frac{5}{2})^2 - 11(\frac{5}{2}) = 4(\frac{5}{2})$$
$$6(\frac{25}{4}) - \frac{55}{2} = \frac{20}{2}$$
$$\frac{75}{2} - \frac{55}{2} = 10$$
$$\frac{20}{2} = 10$$
$$10 = 10$$

Solve each special quadratic equation on a separate sheet of paper. Check your answers. Show your work for the solution and the check. Answers can be found in the end of the packet. For additional help, check out www.MrPelzer.com.

1. $x^2 + 5 = 30$
2. $x^2 - 3 = 125$
3. $2x^2 + 5 = 103$
4. $45x^2 - 586 = 19{,}259$
5. $12x^2 + 420 = 40x^2 - 1372$
6. $4x^2 + 5 = 54$
7. $3x^2 + 12 = 22$
8. $5x^2 + 5 = x^2 + 25$
9. $x^2 + 5x = 0$
10. $2x^2 = 32x$
11. $3x^2 - 6x = 11x$
12. $4x(x - 5) = -2(x^2 + 3x)$

Answer Key

1. Solving Linear Equations p2
1. x = 3
2. x = -7
3. x = 45
4. x = 112/5 = 22.4
5. x = -3
6. x = -9
7. x = 7
8. x = 2/3
9. x = -4/3
10. x = 49/4 = 12.25
11. x = 1/2
12. x = 0

2. Linear Combination (Elimination) p4
1. (4, 2)
2. (5, -1)
3. infinite solution - same line
4. (-2, 12)
5. (912, 873)
6. no solutions - parallel lines
7. (2/3, 3/4)
8. (-7/2, 1/6)
9. (-29/43, -30/43)
10. (-17, 0) – put equations into standard form
11. (-1, 9)
12. (1/2, -3)

3. Substitution p6
1. (12, -3)
2. (-10, 0)
3. infinite solutions - same line
4. (8, -6)
5. (0, 3)
6. (2, 4/3)
7. (-4, 12)
8. (3/2, 12/5)
9. (-1/4, 5/12)

4. Simplify Radicals p8
1. $2\sqrt{10}$
2. $2\sqrt{13}$
3. $4\sqrt{5}$
4. $8\sqrt{5}$
5. $9\sqrt{3}$
6. $12\sqrt{2}$
7. $15\sqrt{3}$
8. 2/3
9. $\sqrt{21}/7$
10. $\sqrt{15}/5$
11. $\sqrt{6}/2$
12. $\sqrt{15}/3$
13. $\sqrt{3}$
14. $2\sqrt{15}/5$
15. 3/2
16. $\sqrt{6}/3$
17. $4\sqrt{6}/3$
18. $5\sqrt{2}/2$

5. Quadratic Formula p10
1. x = -4, -1
2. x = -2, 3
3. x = -6, 0
4. x = 2, 4
5. $x = \dfrac{9 \pm \sqrt{77}}{2} \approx .11, 8.89$
6. x = -5 (double solution)
7. $x = \dfrac{-2 \pm \sqrt{2}}{2} \approx -1.71, -0.29$
8. x = 0.5 (double solution)
9. $x = \dfrac{4 \pm \sqrt{13}}{3} \approx 0.13, 2.54$
10. $a = \dfrac{-3 \pm \sqrt{3}}{3} \approx -1.58, -0.42$
11. $\dfrac{-1 \pm \sqrt{-3}}{2}$ - no real solutions
12. x = -1, 7/4

6. Factor (a = 1) p12
1. x = -3, -1
2. x = -5, 7
3. x = 2, 6
4. x = -2, 12
5. x = -6, 8
6. x = -1, 12
7. x = -22, 2
8. x = -5, 5
9. x = -20, -12

7. Factor (a ≠ 1) p15
1. x = 4/3, 2
2. x = -3/2, 1
3. x = 1, -3/5
4. x = 5/2, 7/2
5. x = 1/3, 4/3
6. x = -2/3, 5
7. x = -9/4, -1
8. x = -1, 3/2
9. x = -1/2, 1

8. Special Quadratics p17
1. $x = \pm 5$
2. $x = \pm 8\sqrt{2}$
3. $x = \pm 7$
4. $x = \pm 21$
5. $x = \pm 8$
6. $x = \pm 7/2$
7. $x = \pm \sqrt{30}/3$
8. $x = \pm \sqrt{5}$
9. x = -5, 0
10. x = 0, 16
11. x = 0, 17/3
12. x = 0, 7/3